which moment the discs appear to attract each other. But if the discs are both charged with the same kind of electricity, the divergence of the electroscopes increases, and at this instant the suspended disc recedes from that which is fixed, being apparently repelled by it.

The author proceeds to examine strictly the nature of this inductive influence, and adduces experiments to render probable that it is in some way dependent on the presence of an exquisitely subtile form of matter which may become disturbed in bodies, and assumes new states or conditions of distribution.

Very numerous experiments are detailed, showing the influence of changes of different intensity, of changes in the dimensions and distances of the opposed discs, of interposed bodies of different forms, &c. on the phenomena of induction. The author concludes by giving the following formulæ as the results of his investigations regarding the elementary laws of electrical induction and attraction. In these expressions Q = quantity of charge, T = the direct induction, q = the quantity of electricity displaced, t = its intensity, T' = the reflected induction, q' = the disturbed quantity, t' = its intensity, q'' = the total quantity in the opposed charged surface, A = the surface, D = the distance between the opposed points, F = the force of attraction.

For the direct induction:

$$T = \frac{Q}{\sqrt{D}} \qquad t = \frac{Q^2}{D}.$$

For the reflected induction:

$$T' = q' = \frac{Q}{D} \qquad t' = \frac{Q^2}{D^2} \qquad q'' = \frac{Q}{\sqrt{D}}.$$

For the attractive force between a charged and a neutral free conductor:

$$F = \frac{Q^2}{D^2} \qquad F = \frac{T}{A^2}.$$

For the force between two unchangeable surfaces, one positive the other negative:

$$F = \frac{Q^2}{D}.$$

2. "On the Conditions of Equilibrium of an Incompressible Fluid, the particles of which are acted upon by Accelerating Forces." By James Ivory, Esq., K.H., M.A., F.R.S., &c.

The intention of this paper is to examine the principles and methods that have been proposed for solving the problem of which it treats, with the view of obviating what is obscure and exceptionable in the investigation usually given of it.

The principle first advanced by Huyghens is clearly demonstrated, and is attended with no difficulty. This principle requires that the resultant of the forces in action at the surface of a fluid in equili-

brium and at liberty, shall be perpendicular to that surface: and it is grounded on this, that the forces must have no tendency to move a particle in any direction upon the surface, that is, in a plane touching the surface.

In the Principia, Sir Isaac Newton assumes that the earth, supposed a homogeneous mass of fluid in equilibrium, has the figure of an oblate elliptical spheroid of revolution which turns upon the less axis: and, in order to deduce the oblateness of the spheroid from the relation between the attractive force of the particles, and their centrifugal force caused by the rotatory velocity, he lays down this principle of equilibrium, that the weights or efforts of all the small columns extending from the centre to the surface, balance one another round the centre. The exactness of this principle is evident in the case of the elliptical spheroid, from the symmetry of its figure: and it is not difficult to infer that the same principle is equally true in every mass of fluid at liberty and in equilibrium by the action of accelerating forces on its particles. In every such mass of fluid, the pressure, which is zero at the surface, increases in descending below the surface on all sides: from which it follows that there must be a point in the interior at which the pressure is a maximum. Now this point of maximum pressure, or centre, is impelled equally in all directions by all the small columns standing upon it and reaching to the surface; and as the pressure in every one of these columns increases continually from the surface to the centre, it follows that the central point sustains the total effect of all the forces which urge the whole body of fluid. It follows also, from the property of a maximum, that the central point may be moved a little from its place without any variation of the pressure upon it: which proves that the forces at that point are zero. Thus the point of maximum pressure is in stable equilibrium relatively to the action of the whole mass of fluid: which establishes Newton's principle of the equiponderance of the central columns in every instance of a fluid in equilibrium and at liberty.

The two principles of Huyghens and Newton being established on sure grounds, the next inquiry is, whether they are alone sufficient for determining the figure of equilibrium. Of this point there is no direct and satisfactory investigation: and, in applying the two principles to particular cases, it has been found that an equilibrium determined by one, is not in all cases verified by the other; and even in some instances, that there is no equilibrium when both principles concur in assigning the same figure to the fluid. Further researches are therefore necessary to dispel the obscurity still inherent in this subject.

In a mass of fluid in equilibrium, if we suppose that small canals are extended from a particle to the surface of the mass, the particle will be impelled with equal intensity by all the canals: for, otherwise, it would not remain immoveable, as an equilibrium requires. It has been inferred that the equal pressures of the surrounding fluid upon a particle are sufficient to reduce it to a state of rest. Hence has arisen the principle of equality of pressure, which is generally

admitted in this theory. Now, if the matter be considered accurately, it will be found that the only point within a mass of fluid in equilibrium which is at rest by the sole action of the surrounding fluid, is the central point of Newton, or the point of maximum-pressure. The reason is that, on account of the maximum, the pressure of all the canals terminating in the central point, increases continually as the depth increases; so that, besides the pressures of the canals, there is no other cause tending to move the particle. With respect to any other particle, the pressure caused by the action of the forces in some of the canals standing upon the particle, will necessarily increase at first in descending below the surface, and afterwards decrease; so that the effective pressure transmitted to the particle, is produced by the action of the forces upon a part only of the fluid contained in such canals. If a level surface be drawn through any particle, it is proved in the paper, that the equal pressures of the surrounding fluid on the particle, are caused solely by the forces which urge the portion of the fluid on the outside of the level surface, the fluid within the surface contributing nothing to the same effect. Thus a particle in a level surface is immoveable by the direct and transmitted action of the fluid on the outside of the level surface; but it will still be liable to be moved from its place unless the body of fluid within the level surface have no tendency to change its form or position by all the forces that act on its own particles.

What has been said not only demonstrates the insufficiency of the principle of equality of pressure for determining the figure of equilibrium of a fluid at liberty, but it points out the conditions which are necessary and sufficient for solving the problem in all cases. The pressure must be a maximum at a central point within the mass: it must be zero at the surface of the fluid: and, these two conditions being fulfilled, there will necessarily exist a series of interior level surfaces, the pressure being the same at all the points of every surface, and varying gradually from the maximum quantity to zero. Now all the particles in the same level surface have no tendency to move upon that surface, because the pressure is the same in all directions: wherefore if we add the condition that every level surface shall have a determinate figure when one of its points is given, it is evident, both that the figure of the mass will be ascertained, and that the immobility of the particles will be established.

Maclaurin's demonstration of the equilibrium of the elliptical spheroid will always be admired, and must be instructive from the accuracy and elegance of the investigation. That geometer was the first who discovered the law of the forces in action at every point of the spheroid; and it only remained to deduce from the known forces the properties on which the equilibrium depends. These properties he states as three in number; and of these, the two which relate to the action of the forces at the surface and the centre of the spheroid, are the same with the principles of Huyghens and Newton, and coincide with two of the conditions laid down above. The third property of equilibrium, according to Maclaurin, consists in this, that every particle is impelled equally by all the rectilineal canals stand-

ing upon it and extending to the surface of the spheroid. Now it does not follow from this property that a particle is reduced to a state of rest within the spheroid, by the equal pressures upon it of the surrounding fluid: because these pressures may not be the effect of all the forces that urge the mass of the spheroid, but may be caused by the action of a part only of the mass. Maclaurin demonstrates that the pressure impelling a particle in any direction is equivalent to the effort of the fluid in a canal, the length of which is the difference of the polar semi-axes of the surface of the spheroid and a similar and concentric surface drawn through the particle, which evidently implies both that the pressures upon the particle are caused by the action of the fluid between the two surfaces, and likewise that the pressures are invariably the same upon all the particles in any interior surface, similar and concentric to the surface of the spheroid. Such surfaces are therefore the level surfaces of the spheroid; and every particle of the fluid is at rest, not because it is pressed equally in all directions, but because it is placed on a determinate curve surface, and has no tendency to move on that surface on account of the equal pressures of all the particles in contact with it on the same surface. Maclaurin seems ultimately to have taken the same view of the matter, when he says that* "the surfaces similar and concentric to the surface of the spheroid, are the level surfaces at all depths." It thus appears that the conditions laid down above as necessary and sufficient for an equilibrium, agree exactly with the demonstration of Maclaurin, when the true import of what is proved by that geometer is correctly understood.

The general conditions for the equilibrium of a fluid at liberty being explained, the attention is next directed to another property, which is important, as it furnishes an equation that must be verified by every level surface. If we take any two points in a fluid at rest, and open a communication between them by a narrow canal, it is obvious that, whatever be the figure of the canal, the effort of the fluid contained in it will be invariably the same, and equal to the difference of the pressures at the two orifices. As the pressure in a fluid in equilibrium by the action of accelerating forces, varies from one point to another, it can be represented mathematically only by a function of three co-ordinates that determine the position of a point: but this function must be such as is consistent with the property that obtains in every fluid at rest. If a, b, c, and a', b', c', denote the co-ordinates of the two orifices of a canal; and $\phi\ (a, b, c)$ and $\phi\ (a', b', c')$ represent the pressures at the same points; the function $\phi\ (a, b, c)$ must have such a form as will be changed into $\phi\ (a', b', c')$, through whatever variations the figure of a canal requires that a, b, c must pass to be finally equal to a', b', c'. From this it is easy to prove that the co-ordinates in the expression of the pressure must be unrelated and independent quantities. The forces in action are deducible from the pressure; for the forces produce the variations of the pressure. As the function that stands for the press-

* Fluxions, § 640.

ure is restricted, so the expressions of the forces must be functions that fulfil the conditions of integrability, without which limitation an equilibrium of the fluid is impossible. Thus, when the forces are given, the pressure may be found by an integration, which is always possible when an equilibrium is possible: and as the pressure is constant at all the points of the same level surface, an equation is hence obtained that must be verified by every level surface, the upper surface of the mass being included. But although one equation applicable to all the level surfaces may be found in every case in which an equilibrium is possible, yet that equation alone is not sufficient to give a determinate form to these surfaces, except in one very simple supposition respecting the forces in action. When the forces that urge the particles of the fluid, are derived from independent sources, the figure of the level surfaces requires for its determination as many independent equations as there are different forces.

In the latter part of the paper the principles that have been laid down are illustrated by some problems. In the first problem, which is the simplest case that can be proposed, the forces are supposed to be such functions as are independent of the figure of the fluid, and are completely ascertained when three co-ordinates of a point are given. On these suppositions all the level surfaces are determined, and the problem is solved, by the equation which expresses the equality of pressure at all the points of the same level surface.

As a particular example of the first problem, the figure of equilibrium of a homogeneous fluid is determined on the supposition that it revolves about an axis and that its particles attract one another proportionally to their distance. This example is deserving of attention on its own account; but it is chiefly remarkable, because it would seem at first, from the mutual attraction of the particles, that peculiar artifices of investigation were required to solve it. But in the proposed law of attraction, the mutual action of the particles upon one another is reducible to an attractive force tending to the centre of gravity of the mass of fluid, and proportional to the distance from that centre: which brings the forces under the conditions of the first problem.

The second problem investigates the equilibrium of a homogeneous planet in a fluid state, the mass revolving about an axis, and the particles attracting in the inverse proportion of the square of the distance. The equations for the figure of equilibrium are two; one deduced from the equal pressure at all the points of the same level surface; and the other expressing that the stratum of matter between a level surface and the upper surface of the mass, attracts every particle in the level surface in a direction perpendicular to that surface. No point can be proved in a more satisfactory manner than that the second equation is contained in the hypothesis of the problem, and that it is an indispensable condition of the equilibrium. Yet, in all the analytical investigations of this problem, the second equation is neglected, or disappears in the processes used for simplifying the calculation and making it more manageable: which is a remarkable

instance of attempting to solve a problem one of the necessary conditions being omitted.

The equations found in the second problem, are solved in the third problem, proving that the figure of equilibrium is an ellipsoid.

3. "Report of a Geometrical Measurement of the Height of the Aurora Borealis above the Earth." By the Rev. James Farquharson, LL.D., F.R.S.

The principal object to which the author directed the inquiries of which he here gives an account, is the determination by geometrical measurement of the height of the aurora borealis, and of the altitude and azimuth of the point to which the streamers seem to converge, and which has been termed the *centre of the corona*: these latter determinations constituting important data for enabling us to form a clear conception of the whole definite arrangement and progress of the meteor, and also a correct judgement of the degree of reliance to be placed on the methods employed for measuring its height above the earth. The paper is chiefly occupied with the details of the observations made or collected by the author, with their critical discussion, with the correction of some misapprehensions which have existed respecting the views stated by the author in his former papers, and with a reply to the strictures of M. Arago on those views.

The result of the geometrical measurement of one particular aurora, gave as the height of its upper edge, 5693 feet above the level of the Manse at Alford; and the vertex of its arch was found to be 14,831 feet northward of the same place. The vertical extension of the fringe of streamers was 3212 feet; leaving 2481 feet for the height of the lower edge above the level of Alford. The tops of the Corean hills, immediately under the aurora, are about 1000 feet higher than that level; so that the lower edge of the arch was only 1500 feet above the summit of that range of hills.

4. "On the Phosphates." By John Dalton, D.C.L., F.R.S., &c.

The author takes a review of the labours of preceding chemists which bear upon the subject of the atomic constitution of phosphoric acid, and the salts in which it enters as a constituent; and shows their conformity with the views he has already advanced on the subject. A supplement is added, giving an account of the effects of various degrees of heat on the salt denominated the *pyrophosphate of soda*.

5. "On the Arseniates." By the Same.

The author here examines the conformity of the results of the analysis of the salts of arsenic with his theory, in the same manner as he has done with the phosphates in the preceding paper.

6. "On the Constitution of the Resins." Parts II. and III. By J. F. W. Johnston, Esq., F.R.S.

In this paper the author, pursuing the train of investigation of